Heat, Mass, and Momentum Transport with Turbulent Flow in Smooth and Rough Pipe

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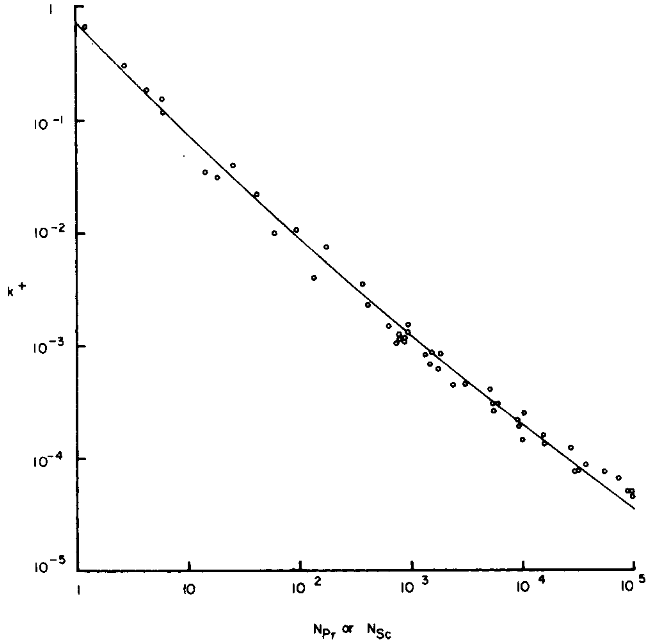

Fig. 1. Laminar sublayer heat and mass transfer coefficient.

A complete model for heat and mass transfer for turbulent pipe flow should be capable of predicting transfer coefficients corresponding to experimental coefficients for the entire range of available Prandtl, Schmidt, and Reynolds numbers represented by these data. Such a model should also be capable of representing the response for a rough pipe as well as a smooth pipe. Friend and Metzner (1958) developed an equation for heat and mass transfer that provides reasonable agreement with experimental data for smooth pipe with Prandtl numbers greater than 0.7. The equation utilizes the shear velocity so that it could be expected to apply to rough surface pipe. The equation is not applicable to systems in which most of the resistance to transfer is in the turbulent core, such as liquid metals. A prior paper by the author (1971) proposed a combination of equations for the wall region and the turbulent core to represent the entire range of experimental Prandtl and Schmidt numbers. An empirical model was used for the wall region. Succeeding notes (1972, 1973) revised the equation for eddy diffusion in the turbulent core and showed that the penetration model is applicable to the transition region and is consistent with the contact time corresponding to the mean period of fluctuations near the wall:

$$\frac{u^{*2}\,t}{\nu} = 338 \tag{1}$$

An additional note (1975) extended this approach to heat transfer with variable viscosity in the wall region. This work shows that the wall region consists of a developing laminar layer at the wall and a transition region that have quite different characteristics. Thus, the empirical wall region representation of the earlier model should be replaced with equations that represent the two distinct divisions of the wall region. This note provides this revision of the earlier model and shows that the resulting model is applicable to the range of experimental data for a smooth and moderately rough pipe.

The three regions of turbulent pipe flow with molecular and eddy diffusion properties result in a three-resistance model:

$$\frac{1}{k^+} = \frac{1}{k^+_{BM} + k^+_{BE}} + \frac{1}{k^+_{TM} + k^+_{TE}} + \frac{1}{k^+_{CM} + k^+_{CE}} \tag{2}$$

with the assumption that the wall region thickness is very small in comparison to the pipe radius so that the area terms are essentially equal and cancel. Assumption of $y^+ = 1.6$ as the limit of the laminar sublayer and $y^+ = 34.6$ for the transition region in accordance with the observations of Popovich and Hummel (1967) with the equations from the prior work yields

$$\frac{1}{k^+} = \frac{1}{k^+_B} + \frac{1}{\dfrac{1}{33\,N_{Sc}} + 0.0615\,N_{Sc}^{-1/2}} + \frac{1}{\dfrac{3.58\,\mathcal{D}}{R\,u^*} + 2\sqrt{f/2}} \tag{3}$$

for mass transfer. The molecular diffusion term for the core applies for a circular pipe. If this core diffusion resistance is negligible, Equation (3) is applicable to other pipe cross sections. Experimental heat and mass transfer data for smooth pipe are available for the Prandtl and Schmidt number range of 0.02 to 100 000. Analysis of the data with Equation (3) shows that the laminar sublayer resistance is negligible in comparison to the transition and core resistances for Prandtl numbers less than unity. The experimental data of Bernardo and Eian (1945), Dipprey and Sabersky (1963), Friend and Metzner (1958), Gowan and Smith (1968), Harriott and Hamilton (1965), Hubbard and Lightfoot (1966), and Mizushina et al. (1971) were used with Equation (3) to calculate the sum of the molecular and eddy coefficients for the laminar sublayer. The results are shown by Figure 1, and the equation

$$k^+_B = \frac{1}{1.6\,N_{Sc}} + 0.062\,N_{Sc}^{-2/3} \tag{4}$$

provides the correlation shown by the figure.

Table 1 shows selected experimental heat and mass transfer data compared with values calculated from Equations (3) and (4) and from the Friend and Metzner (1958) equation. Equation (4) represents the electro-

TABLE 1. COMPARISON OF EXPERIMENTAL AND CALCULATED NUSSELT NUMBERS

Data source	N_{Pr} or N_{Sc}	N_{Re}	N_{Nu} Experimental	N_{Nu} Equations (3) and (4)	N_{Nu} F and M
Dipprey and Sabersky (1973)	1.2	520,000	1,030	1,020	980
	5.94	30,000	195	189	175
Friend and Metzner (1958)	59	12,000	240	242	275
	370	8,100	350	315	300
Gowen and Smith (1968)	14.3	30,000	248	280	260
Harriott and Hamilton (1965)	97,600	10,000	2,500	1,880	2,470
Kolar (1965)	0.7	28,500	67	67	74
Mizushina et al. (1971)	631	32,500	638	645	960
	15,100	15,500	1,470	1,550	1,920
Sleicher et al. (1973)	0.0203	26,000	6.8	8.7	—
	0.0245	302,000	29	32.2	—

chemical rather than wall solution data at high Schmidt numbers because of the possibility of increased surface with the solution method. Thus, good agreement between Equations (3) and (4) and the Mizushina et al. data is observed with Table 1 and rather poor agreement for the Harriott and Hamilton data. The Friend and Metzner equation is shown by Table 1 to agree with the wall solution data rather than the electrochemical data.

Both models are observed to provide reasonable estimates of the transfer coefficients except for liquid metals, where the Friend and Metzner equation is not applicable.

ROUGH SURFACES

The increased shear stress from rough pipe surfaces could be expected to correspond to the wall region fluctuations in accordance with Equation (1). Thus, Equations (3) and (4) should be applicable for turbulent flow with a rough surface. Figure 2 shows a comparison of calculated and experimental heat transfer coefficients from the data of Dipprey and Sabersky, Gowen and Smith, and Kolar. These data include a sand grain roughness and roughness formed by cutting triangular threads in the tube wall. Data shown represent moderate roughness. Experimental coefficients are much less than calculated coefficients at high Reynolds numbers and extreme roughness. This may occur when the wall surface roughness dimension is large in comparison to the turbulent flow wall region thickness. The data shown by Figure 2 represent a dimensionless roughness ($\epsilon u^*/\nu$) of less than 100. Data for air show agreement of calculated and experimental coefficients at greater dimensionless roughness values than 100, probably because the wall region represents less contribution with Equation (3) than occurs with water. Coefficients calculated by the Friend and Metzner equation are essentially the same as those calculated by Equations (3) and (4) for the data shown by Figure 2 for values of the Prandtl number greater than unity. Calculated coefficients from the Friend and Metzner equation for air show poor agreement with experimental data, probably because of the large core contribution for these data.

In summary, a heat and mass transfer model is presented which provides good agreement with experimental data for turbulent pipe flow over the available range of Prandtl, Schmidt, and Reynolds numbers. The model also shows agreement with data for pipe with moderate roughness.

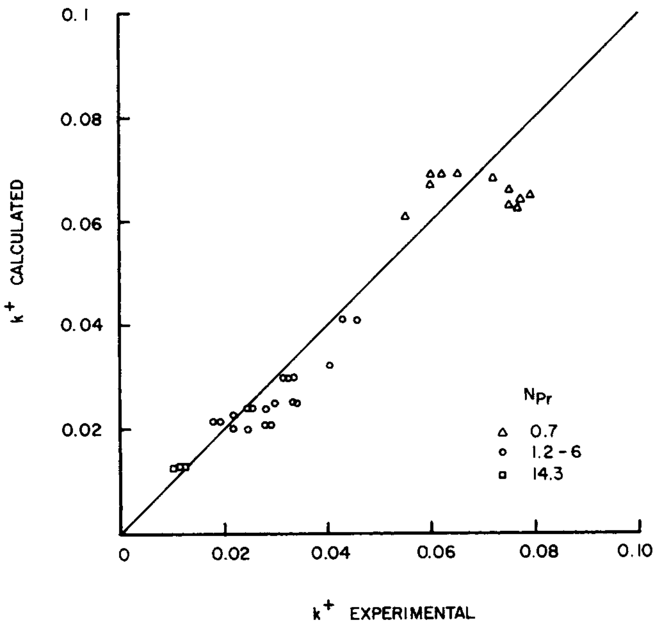

Fig. 2. Rough pipe heat transfer data

NOTATION

C_p = specific heat
$\mathcal{D}$ = molecular diffusivity
f = friction factor
k^+ = dimensionless transfer coefficient, k/u^* or $h/\rho C_p u^*$
N_{Pr} = Prandtl number
N_{Re} = Reynolds number
N_{Sc} = Schmidt number
R = core radius for circular pipe
t = eddy contact time
u^* = shear velocity
y^+ = $y/u^*\nu$
ϵ = equivalent roughness
ν = kinematic viscosity
ρ = density

Subscripts

B = laminar sublayer
BE = laminar sublayer, eddy diffusion
BM = laminar sublayer, molecular diffusion
CE = core, eddy diffusion
CM = core, molecular diffusion
TE = transition region, eddy diffusion
TM = transition region, molecular diffusion

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Transfer Coefficient in Pulsating Pipe Flow: Comments on an Article by Patel et al.

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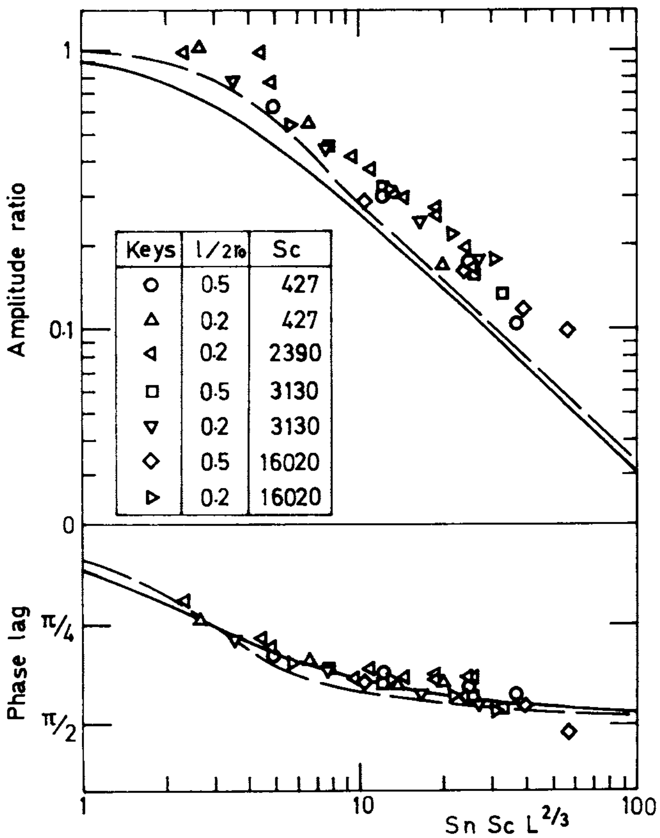

Fig. 1. Bode diagram of space averaged Sherwood number. ——, Equation (14); - -, Equation (5) in the paper by Mizushina et al. (1973).

In their paper on measurements of wall mass transfer in fully developed pulsating laminar flow in a tube, Patel et al. (1975) state that at higher frequencies the amplitude and phase of the wall mass transfer rate are correlated not by the theoretically derived variable $SnScL^{2/3}$, but by an empirical correlation variable $SnScL^{1/3} \times (l/2r_0)^{1/3}$. At the same time, they suggest some possible sources of error in both their experimental and theoretical treatments, but they leave the questions to be unsolved, stating that "the present work reports the first experimental measurements of amplitude and phase of the fluctuating mass transfer coefficient in pulsatile flow."

However, this is not the case. The experimental frequency response of the space averaged, mass transfer coefficient was already reported by us (Mizushina et al., 1973). We made measurements over wide ranges of Re, Sc, and Sn, using the electrochemical method. Figure 1 shows the frequency response of the space averaged value to the wall shear stress. As can be seen, the measurements are well correlated by the theoretically derived variable $SnScL^{2/3}$ and in good agreement with our theoretical curves. In addition, the validity of the correlation by the theoretically derived variable is confirmed for the local transfer coefficient under the heating condition of constant wall heat flux.

On the basis of this fact, we will make some comments on the empirical correlation variable proposed by Patel et al. (1975) and on their experimental procedure. The variable $SnScL^{2/3}$ shows dependences on Sc and Re as $Sc^{1/3}$ and $Re^{-2/3}$, while the empirical correlation variable $SnScL^{1/3}$ $(l/2r_0)^{1/3}$ has different dependences, that is, $Sc^{2/3}$ and $Re^{-1/3}$. However, the necessity of the 2/3 exponent of Sc instead of the 1/3 exponent is not clear,